this time: a case study; experiments, events, sample space, outcomes/elements, set theory

next time:

read: DeGroot & Schervish (DS) ch. 1

AMS 131

25 Jul 2016

① doc. cam. notes

course web site

ams131-summer16-01.courses.soe.ucsc.edu

the course is being video-recorded; please go to webcast.ucsc.edu to see the videos

TAs: Catherine Brennan, Cheng-Han Yu

book: DeGroot & Schervish

Instructor: David Draper, draper@ucsc.edu, Baskin 357C

email me if you want to add the class: put AMS 131 student in subject line

population sample

9.56 ②

$$\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix} \xrightarrow[\text{random}]{\text{at}} \begin{bmatrix} Y_1 \end{bmatrix} \quad n = 1$$

ELM?
yes, from meaning of "at random"

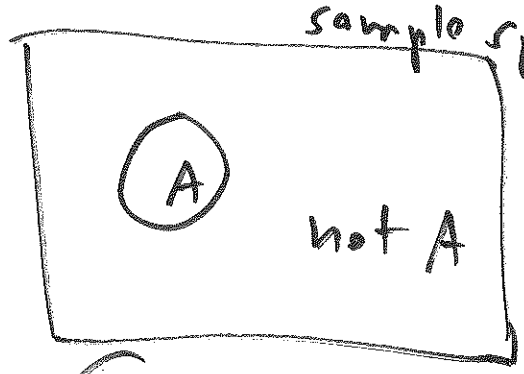
$$P(Y_1 \text{ is odd}) = \frac{2}{3}$$

"classical" approach → PF Pascal - Fermat

$\boxed{\text{not}}$

$$P(A) = \frac{\text{area of } A}{\text{area of } \square}$$

sample space



← 1

$\boxed{\text{easy rule}}$ ① for any <u>event</u> A,

$$0 = 0\% \le P(A) \le 1 = 100\%$$

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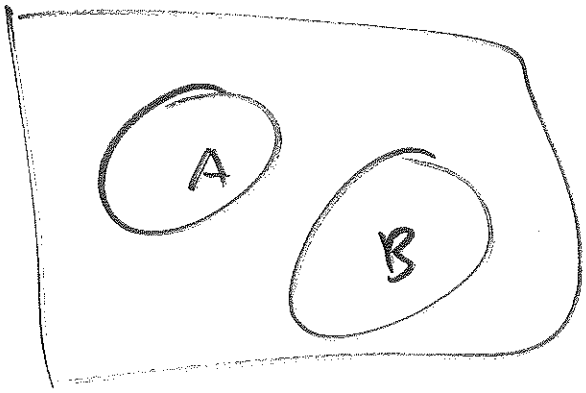
② $P(A) + P(\text{not } A) = 1$

→ $P(A) = 1 - P(\text{not } A)$

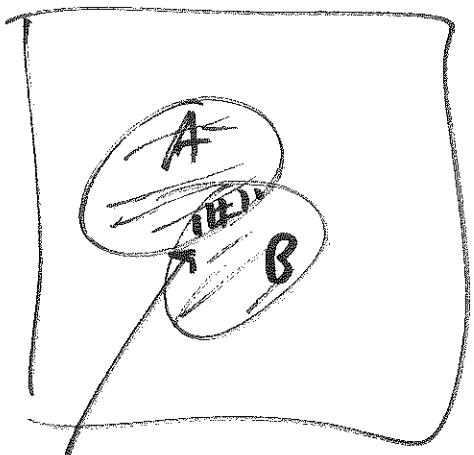
directly } indirect

or



no overlap: mutually exclusive

$$P(A \text{ or } B) = P(A) + P(B)$$



(A and B)

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

general addition rule for or

and

population $\begin{bmatrix} 1 \\ 2 \\ 9 \end{bmatrix}$ $\xrightarrow[\text{random}]{\text{at}}$ sample $\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}$ $n = 2$

either at random with replacement
(independent identically distributed)
(IID) sampling

or at random without replacement
(simple random sampling (SRS))

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with replacement

draw 2 (Y_2)

draw 1 (Y_1)	1	2	9
1	(1,1)	(1,2)	(1,9)
2	(2,1)	(2,2)	(2,9)
9	(9,1)	(9,2)	(9,9)

ELM?

yes: all 9 possibilities equally likely

$$P(\overbrace{Y_1 = 9}^{A} \text{ and } \overbrace{Y_2 = 9}^{B}) = \frac{1}{9} \text{ by ELM}$$

$$\boxed{P(\underset{A}{Y_1 = 9}) = \frac{3}{9} = \frac{1}{3}} \qquad P(\underset{B}{Y_2 = 9}) = \frac{1}{3}$$

theory: $P(A \text{ and } B) = P(A) \cdot P(B)$

without replacement

		Y_2: 1	2	9
Y_1	1	~~(1,1)~~	(1,2)	(1,9)
	2	(2,1)	~~(2,2)~~	(2,9)
	9	(9,1)	(9,2)	~~(9,9)~~

ELM? Yes: all 6 possibility equally likely

$$\boxed{P(Y_1 = 9) = \frac{1}{3} = \frac{2}{6}} \qquad P(Y_2 = 9) = \frac{2}{6} = \frac{1}{3}$$

$$P(Y_1 = 9 \text{ and } Y_2 = 9) = 0 \neq \frac{1}{3} \cdot \frac{1}{3}$$

theory fails when Y_2 depends on Y_1

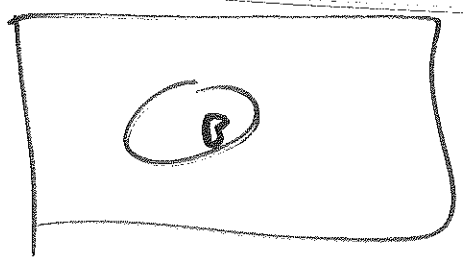
conditional probability

Rev. Bayes (1740)

$P(B \text{ given } A) = ?$

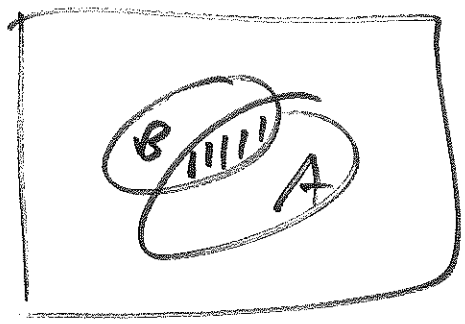
$= P(B \mid A)$

$$P(B) = \frac{B}{\square}$$



definition

$$P(B \mid A) = \frac{A \text{ and } B}{A}$$



$$= \frac{P(B \text{ and } A)}{P(A)} = \frac{P(A \text{ and } B)}{P(A)}$$

$$P(B \mid A) = \frac{P(A \text{ and } B)}{P(A)}$$ multiply by $P(A)$

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

general product rule for and

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)} \quad \text{multiply by } P(B)$$

$$P(A \text{ and } B) = P(B) \cdot P(A \mid B)$$

$A = (Y_1 = 9)$

$B = (Y_2 = 9)$

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

$$= \frac{1}{3} \cdot 0 = 0 \checkmark$$

with replacement:

1st draw, 2nd draw are independent:

$\boxed{\text{definition}}$ A, B independent $\overset{\text{iff}}{\Longleftrightarrow}$

information about A doesn't change chances for B & vice versa

(with repl.)

$$P(Y_2 = 9 \mid Y_1 = 9) = P(Y_2 = 9)$$

A, B independent $\leftrightarrow$ $P(B \mid A) = P(B)$

and $P(A \mid B) = P(A)$

with repl.

$$P(Y_1 = 9 \text{ and } Y_2 = 9) =$$

$$P(Y_1 = 9) \cdot P(Y_2 = 9 \mid Y_1 = 9)$$

$$\overset{\text{indep}}{=} P(Y_1 = 9) \cdot P(Y_2 = 9) \quad \checkmark$$

general product rule for and

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$
$$= P(B) \cdot P(A \mid B)$$

if A, B indep.

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

P(1 or more T-s in family of 5)

$$= P\left(\begin{matrix}\text{exactly}\\ 1\\ \text{T-s}\end{matrix} \text{ or } \begin{matrix}\text{exactly}\\ 2\end{matrix} \text{ or } \cdots \text{ or } \begin{matrix}\text{exactly}\\ 5\end{matrix}\right)$$

$$= P\left(\begin{matrix}\text{exactly}\\ 1\\ \text{T-s}\end{matrix}\right) + P\left(\begin{matrix}\text{exactly}\\ 2\end{matrix}\right) + \cdots + P\left(\begin{matrix}\text{exactly}\\ 5\end{matrix}\right)$$

direct

$$P(1 \text{ or more}) = 1 - P\left(\begin{matrix}\text{exactly 0}\\ \text{T-s}\end{matrix}\right)$$

$$= 1 - P\left(\begin{matrix}\text{not}\\ \text{T-s}\\ \text{on 1st}\end{matrix} \text{ and } \begin{matrix}\text{not}\\ \text{T-s}\\ \text{on 2nd}\end{matrix} \text{ and } \cdots \text{ and } \begin{matrix}\text{not}\\ \text{T-s}\\ \text{on 5th}\end{matrix}\right)$$

indep

$$= 1 - P\left(\begin{matrix}\text{not}\\ \text{T-s}\\ \text{on 1st}\end{matrix}\right) \cdot P\left(\begin{matrix}\text{not}\\ \text{T-s}\\ \text{on 2nd}\end{matrix}\right) \cdots P\left(\begin{matrix}\text{not}\\ \text{T-s}\\ \text{on 5th}\end{matrix}\right)$$

$$= 1 - \left[1 - P\left(\begin{matrix}\text{T-s}\\ \text{body}\\ \text{on 1st}\end{matrix}\right)\right] \cdots \left[1 - P\left(\begin{matrix}\text{T-s}\\ \text{on}\\ \text{5th}\end{matrix}\right)\right]$$

$$\doteq 76\%$$

indep. dist

$$= 1 - \left(1 - \frac{1}{4}\right) \cdots \left(1 - \frac{1}{4}\right) = 1 - \left(1 - \frac{1}{4}\right)^5$$

(10.54)

bogus ELM argument: ① there are 32 possible outcomes in S; ② I assert that ELM applies to this enumeration; ③ $P(1 \text{ or more } T) = \frac{31}{32}$

bogus because ELM doesn't apply: N N N N N is more likely than (ex.) T N N N N