This is a case study; next time: experiments, space, outcomes/elements, course website.

Set theory

AMS131 - Summer 16-01 course.

The course is being video-recorded; please go to webcast.ucsc.edu to see the videos.

TAs: Catherine Brennan
Cheng-Han Yu

Instructor: David Draper

*Email me if you want to add the class subject line*

Red: DeGroot

Schervish

(DS) ch. 1 doc. can be seen

AMS 131

25 Jul 2016
Population sample

\[
\begin{bmatrix}
1 \\
2 \\
9
\end{bmatrix} \overset{\text{random}}{\rightarrow} \begin{bmatrix} Y_1 \end{bmatrix}, n = 1
\]

\[p(\text{if } Y_1 \text{ is odd}) = \frac{2}{3}\]

Classical approach: Pascal - Fermat

Sample space

\[p(A) = \frac{\text{area of } A}{\text{area of } \Omega}\]

easy rule

\[0 \leq p(A) \leq 1 = 100\%\]

\[p(A) + p(\text{not } A) = 1\]

\[\Rightarrow p(A) = 1 - p(\text{not } A)\]

directly 1 indirectly
\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]

General addition rule for \( \text{or} \)
population
\[
\begin{bmatrix}
1 \\
2 \\
9
\end{bmatrix}
\]

random

either

at random with replacement (independent identically distributed (IID) sampling)

or

at random without replacement (simple random sampling (SRS))

with replacement

\[
\begin{array}{ccc}
& 1 & 2 & 9 \\
1 & (1,1) & (1,2) & (1,9) \\
2 & (2,1) & (2,2) & (2,9) \\
9 & (9,1) & (9,2) & (9,9)
\end{array}
\]

draw 1

\(1\)

draw 2

\(9\)

ELM?

\(\begin{array}{c}
1 \quad 2 \quad 9 \\
1 \quad (1,1) \quad (1,2) \quad (1,9) \\
2 \quad (2,1) \quad (2,2) \quad (2,9) \\
9 \quad (9,1) \quad (9,2) \quad (9,9)
\end{array}\)

\(\text{Yes: all 9 possibilities equally likely}\)
\[ P(\bar{y}_1 = 9 \text{ and } y_2 = 9) = \frac{1}{9} \text{ by ELM} \]

\[ P(\bar{y}_1 = 9) = \frac{3}{9} = \frac{1}{3} \]

\[ P(\bar{y}_2 = 9) = \frac{1}{3} \]

**Theory:** \[ P(A \text{ and } B) = P(A) \cdot P(B) \]

Without replacement:

\[
\begin{array}{c|c|c|c|c|c|c|c}
 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
1 & (1, 1) & (1, 2) & (1, 3) & (1, 4) & (1, 5) & (1, 6) \\
2 & (2, 1) & \times & \times & (2, 3) & \times & \times \\
3 & \times & \times & \times & (3, 4) & \times & \times \\
4 & \times & \times & \times & \times & \times & \times \\
5 & \times & \times & \times & \times & \times & \times \\
6 & \times & \times & \times & \times & \times & \times \\
\end{array}
\]

**ELM:**

\[ \begin{align*}
\text{P}(\bar{y}_1 = 9, y_2 = 9) &= \left( \frac{1}{3} \times \frac{1}{6} \right) = \frac{1}{18} \\
\text{P}(\bar{y}_1 = 9) &= \frac{1}{3} \\
\text{P}(y_2 = 9) &= \frac{1}{6} \\
\text{P}(\bar{y}_1 = 9 \text{ and } y_2 = 9) &= 0 \neq \frac{1}{9} \frac{1}{3} \\
\end{align*} \]

Theory fails when \( y_2 \) depends on \( y_1 \).
Conditional probability

Rev. Bayes (1740)

\[ P(B \text{ given } A) = \frac{P(B \cap A)}{P(A)} \]

Definition

\[ P(B \mid A) = \frac{P(A \cap B)}{P(A)} \]

\[ P(B \mid A) = \frac{P(A \cap B)}{P(A)} \]

\[ P(A \cap B) = P(A) \cdot P(B \mid A) \]
The general product rule for $A$ and $B$ is:

$$P(A \land B) = \frac{P(A \land B)}{P(B)} \cdot P(A|B)$$

$p(A \land B) = P(B) \cdot P(A|B)$

$A = (\gamma_1 = 9)$

$B = (\gamma_2 = 9)$

$$p(A \land B) = p(A) \cdot p(B|A)$$

with replacement:

$$\frac{1}{3} \cdot 0 = 0 \checkmark$$

1st row, 2nd row are independent:

**Definition**: $A, B$ independent $\iff$ information about $A$ doesn't change chances for $B$ & vice versa.
\[ p(y_2 = 9 \mid y_1 = 9) = p(y_2 = 9) \]

A, B independent \iff \[ p(B \mid A) = p(B) \]
and \[ p(A \mid B) = p(A) \]

\[ p(y_1 = 9 \text{ and } y_2 = 9) = p(y_1 = 9) \cdot p(y_2 = 9 \mid y_1 = 9) \]

\[ = p(y_1 = 9) \cdot p(y_2 = 9) \quad \checkmark \]

General product rule for \( \text{and} \)

\[ p(A \text{ and } B) = p(A) \cdot p(B \mid A) = p(B) \cdot p(A \mid B) \]

if A, B independent.

\[ p(A \text{ and } B) = p(A) \cdot p(B) \]
\[ P(1 \text{ or more } T-S \text{ in family of } 5) \]
\[ = P(\text{exactly } 1 \text{ or exactly } 2 \text{ or } \ldots \text{ or exactly } 5) \]
\[ = P(\text{exactly } 1 \text{ or exactly } 2) + P(\text{exactly } 3) + \ldots + P(\text{exactly } 5) \]
\[ \text{direct} \]
\[ P(1 \text{ or more}) = 1 - P(\text{exactly } 0) \]
\[ = 1 - P(\text{not } + \text{ T-S or 1ST or 2nd or 3rd or 4th or 5th}) \]
\[ \text{indep} \]
\[ = 1 - [1 - P(\text{not } + \text{ T-S or 1ST}) \cdot P(\text{not } + \text{ T-S or 2nd}) \ldots P(\text{not } + \text{ T-S or 5th})] \]
\[ \geq \text{indep} \]
\[ = 1 - (1 - \frac{1}{4}) \cdot \ldots \cdot (1 - \frac{1}{4}) \cdot 1 - (1 - \frac{1}{4}) \]
\[ \geq \frac{76\%}{10.54} \]
1) 32 possible outcomes in $S$;
2) I assert that ELM applies to this enumeration; 3) $P(1$ or more$) = \frac{31}{32}$

[Circle indicates erroneous choice: because ELM doesn't apply: NNNNNN]

is more likely than

(ex.) TNNNNN