This dist. of time: functions of rvs, expectation.

next expectation time: of sums and products, variance, SD; other moments of the dist.

real: DS ch. 4 pp. 215-260

Case study: option pricing

Silicon valley companies give signing bonuses as incentives to accept their job offers.

They are often in the form of stock options: an opportunity to buy N shares of the (known) company one year from now at a price.

In your view, if the stock is likely to rise over the next year, you'll be able to sell at a profit. Define \( X = (\text{price of the stock 1 year from now}) \).

For simplicity, pretend \( X \) is discrete
with only 2 values: \( x_1 < S \) and \( x_2 > S \). Let \( P = \mathbb{P}(X = x_2) \), the prob.
that the stock will rise in value. You'd
like to evaluate these stock options
(e.g., to compare one company's job offer
with that of another), but (of course)
you don't know \( X \). Let \( V = \) value of option
for one share at \$S \) 1 year from now.
If \( (X = x_1 < S \) ), the option is worthless
and \( V = 0 \); otherwise (ignoring dividends
and costs of buying & selling stocks) if
\( (X = x_2 > S \) ) then the option is worth
\( (x_2 - S) \); thus \( V = h(X) = \begin{cases} 0 & \text{if } X = x_1 \\ (x_2 - S) & \text{if } X = x_2 \end{cases} \)

To see how valuable
the option is, you have to compare
it to the return you would have received if you had not 'exercised the stock option'; a reasonable point of comparison would be to invest in a bond that pays a% per year.

A fair measure of worth of the option would be the present value of \( I \), defined to be the number \( c \) such that

\[
E(I) = (1+d) \cdot c.
\]

But we already know that

\[
E(I) = 0 \cdot (1-p) + (x_2 - \delta) \cdot p = (x_2 - \delta) p,
\]

so \((1+d) \cdot c = (x_2 - \delta) p\) and \(c = \left( \frac{x_2 - \delta}{1+d} \right) p\).

to finish the calculation you need to specify \( p \). The standard way to do this...
in the financial sector is to assume that the present value of $X$ is equal in expectation to the current value of the stock price: i.e., to assume that the expected value of (buying 1 share & holding it for a year) = (investing the same amount of money in the risk-free alternative) – i.e., $E(X) = (1 + \alpha) \cdot S'$.

But we already know that

$$E(X) = p \cdot x_2 + (1 - p) \cdot x_1 = (1 + \alpha) S'$$

so solving for $p$ gives

$$p = \frac{x_1 - (1 + \alpha) S'}{x_1 - x_2} = \frac{(1 + \alpha) S' - x_1}{x_2 - x_1}.$$

So the fair price of an option to buy
one share is given by: $c = \left( \frac{x_t - x^*}{x_t - x_1} \right) \left[ \frac{(1 + 0.05)(x^* - x)}{1 + d} \right].$

If we use as illustration $S = 200$

$x_1 = 180$

$x_2 = 260$

$\sigma = 0.04$ as realistic in 2001 or so but not today: $d = 0.01$ now

With these values $c = \$20,$

(about 10% of the current value of the stock).

$c$ is called the risk-neutral price of the option; under the assumptions made here, you could now sell the option today (if you had it) at a fair price of about $\$20$; this would make you an options trader.

An investment that allows people to buy or sell an option on a security is called a derivative.
Examples of invertible (1-1) and differentiable functions.
mean is pulled by the tail

mean much more influenced by outliers than median

unsurprising real estate person: quote median to a buyer but mean to a seller

you: ask for both mean & median.