AMS 131
10 Aug 16

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(com. notes)

this: dist. of time: functions of rvs, expectation

read: DS ch. 4 pp. 215–260

Case study: option pricing

next: expectation time: of sums and products, variance, SD; other moments of the dist.

Silicon Valley companies give <u>signing bonuses</u> as incentives to accept their job offers. These are often in the form of <u>stock options</u>: an opportunity to buy N shares of the company one year from now at a (known) price,

S. If the stock is likely to rise over the next year, you'll be able to sell at a profit.

in your view

Define X = (price of the stock 1 year from now).

For simplicity pretend X is discrete

with only 2 values: $x_1 < S$ and $x_2 > S$, let $p = P(X = x_2)$, the prob. that the stock will rise in value. You'd like to evaluate these stock options (e.g., to compare one company's job offer with that of another), but (of course) you don't know X. Let Y = value of option for one share at S 1 year from now.

If $(X = x_1 < S)$, the option is worthless and $Y = 0$; otherwise (ignoring dividends & costs of buying & selling stocks) if $(X = x_2 > S)$ then the option is worth $(x_2 - S)$; thus

$$Y = h(X) = \begin{cases} 0 & \text{if } X = x_1 \\ (x_2 - S) & x_2 \\ 0 & \text{else.} \end{cases}$$

To see how valuable the option is, you have to compare

it to the return you would have received if you had not "exercised the stock option"; a reasonable point of comparison would be to invest in a bond or other fixed security that pays α%/year.

A fair measure of worth of the option would be the <u>present value</u> of Y, defined to be the number c such that

$$E(Y) = (1+\alpha)\cdot c.$$

But we already know that $E(Y) = 0\cdot(1-p) + (x_2 - S')\cdot p = (x_2 - S')p$,

so $(1+\alpha)\cdot c = (x_2 - S')\cdot p$ and $c = \left(\frac{x_2 - S'}{1+\alpha}\right)\cdot p$.

To finish the calculation you need to <u>specify p</u>. The standard way to do this

in the financial sector is to assume that (*) the present value of $\underline{X}$ is equal in expectation to the current value of the stock price: i.e., to assume that the expected value of (buying 1 share & holding it for a year) = (investing the same amount of money in the risk-free alternative) — i.e., $E(\underline{X}) = (1+\alpha)\cdot S$.

But we already know that

$E(\underline{X}) = p\cdot x_2 + (1-p)\cdot x_1 \triangleq (1+\alpha)S$; solving for p gives $p = \dfrac{x_1 - (1+\alpha)S}{x_1 - x_2} = \dfrac{(1+\alpha)S - x_1}{x_2 - x_1}$.

So the fair price (c) of an option to buy

one share is given by

$$c = \left(\frac{x_2 - S}{1+\alpha}\right)\left[\frac{(1+\alpha)S - x_1}{x_2 - x_1}\right].$$

DS use as illustration

$S = 200$

$x_1 = 180$

$x_2 = 260$

$\alpha = .04$ realistic in 2001 or so but not today: $\alpha \doteq .01$ now

downside \$20 (-10%)

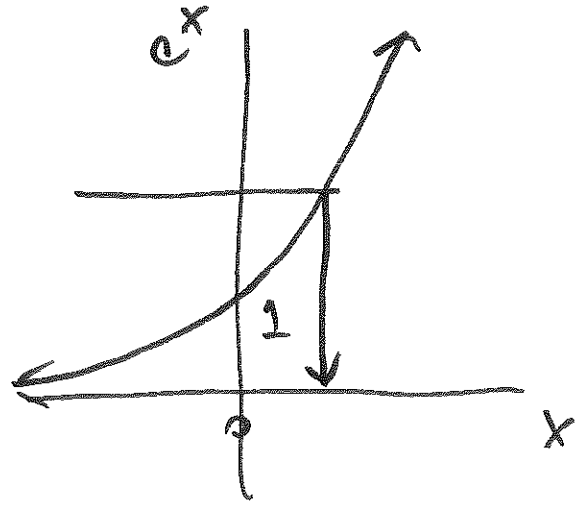
upside \$60 (+30%)

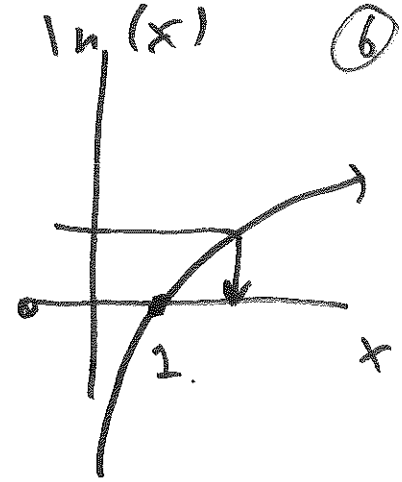
With these values $c \doteq \$20$ (about 10% of the current value of the stock).

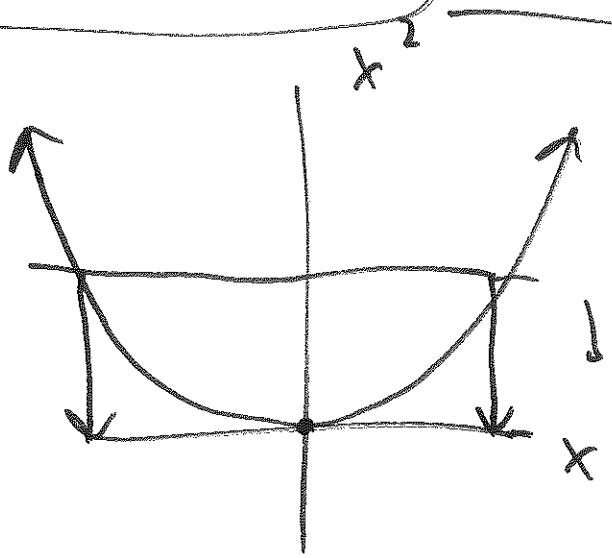
c is called the <u>risk-neutral price</u> of the option; under the assumptions made here, you could now sell the option today (if you had it) at a fair price of about \$20; this would make you an <u>options trader</u>.

An investment that allows people to buy or sell an option on a security (e.g. stock) is called a <u>derivative</u>.

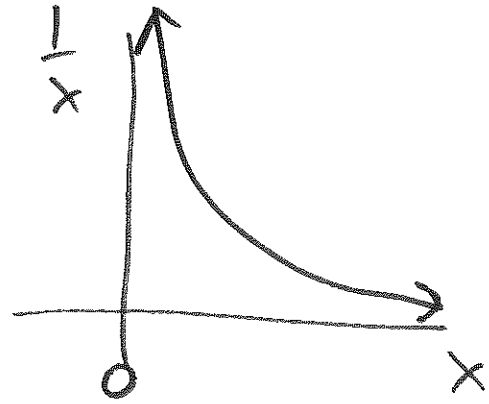
Examples of invertible (1-1) & differentiable functions

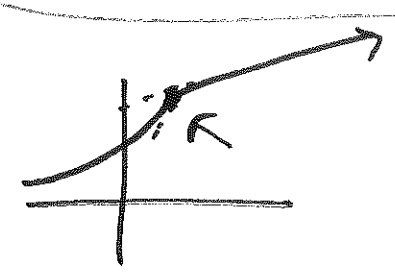


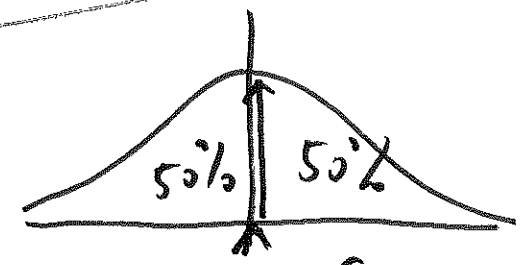




diff ✓
but not
1-1





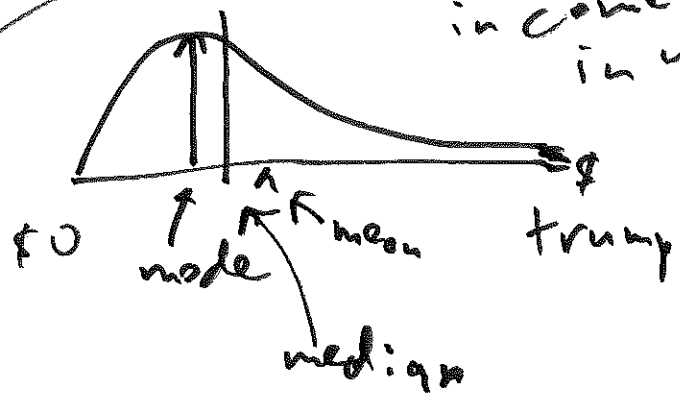


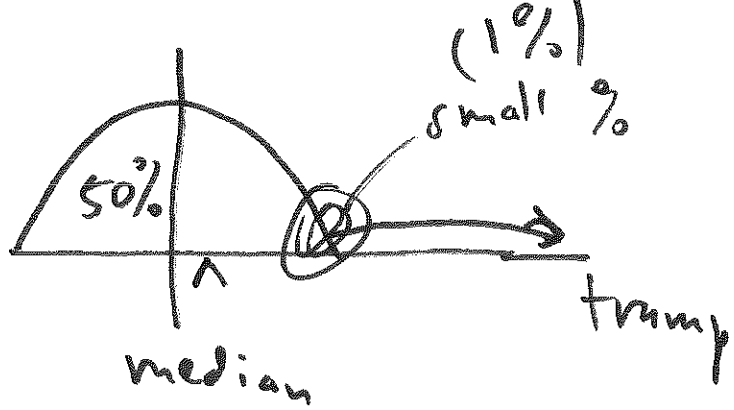
point of symmetry
= mode
= median
= mean

symmetric
& unimodal

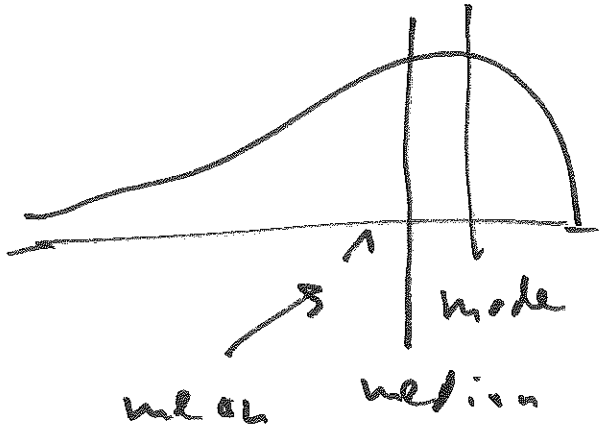
(9.50)

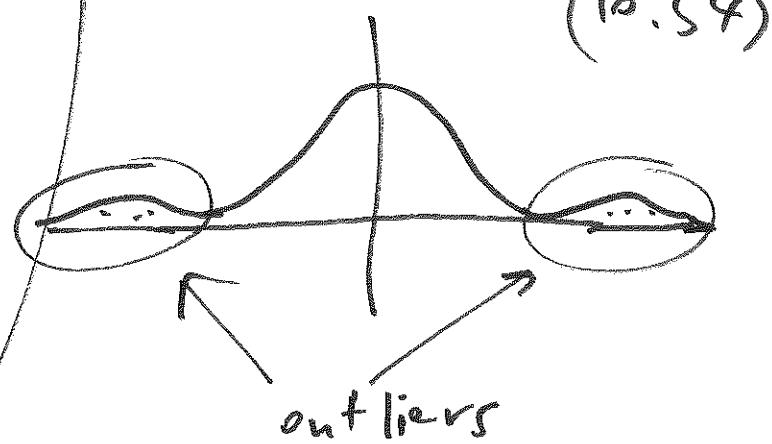
annual family income in U.S.



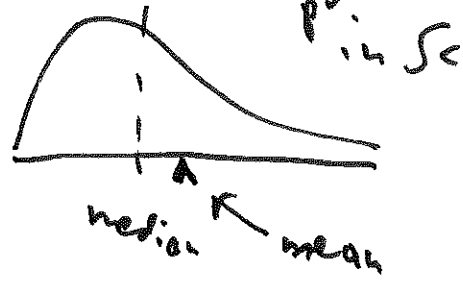


mean is pulled by the tail





mean much more influenced by outliers than median



unscrupulous real estate person: quote median to a buyer but mean to a seller

you: ask for both mean & median