this time: Δ-method, continuity correction, Markov chains

next time: Markov chains

read: DS pp. 188-201

AMS131, 24 Aug 2016

(doc. ① con. notes)

$$E(S) = E\left(\sum_{i=1}^{n} W_i\right)$$

$$= \sum_{i=1}^{n} E(W_i)$$

$$= \sum_{i=1}^{n} \mu = \boxed{n\mu = E(S)}$$

$$= 30{,}336 \text{ lb.}$$

$$SE(S) = \sqrt{V(S)}$$

↑ standard error

$$= \sqrt{V\left(\sum_{i=1}^{n} W_i\right)}$$

$$\overset{\text{IID}}{=} \sqrt{\sum_{i=1}^{n} V(W_i)}$$

$$\overset{\text{IID}}{=} \sqrt{\sum_{i=1}^{n} \sigma^2}$$

$$= 33 \text{ lb}\sqrt{192}$$

$$\doteq 457 \text{ lb}$$

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} W_i$$

$$SE(\bar{X}_n) = \frac{\sigma}{\sqrt{n}}$$

$$SE(S) = \sqrt{n\sigma^2} = \sigma\sqrt{n}$$

(10.00) The confidence interval story

Neyman (~1930)

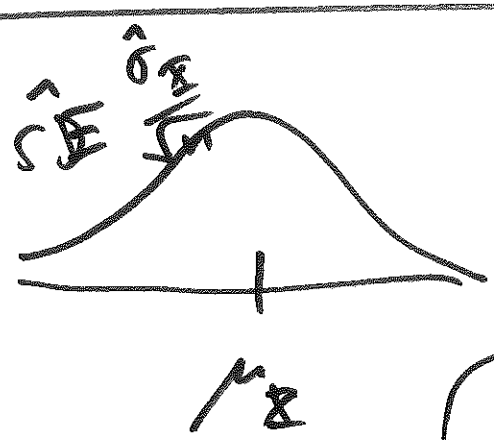
single most important frequentist inferential idea

$(\sigma_X^2 < \infty)$

$X_1, \ldots, X_n \overset{IID}{\sim}$ some dist.

$E(X_i) = \mu_X$, $V(X_i) = \sigma_X^2$

for n large,



dist (PDF) of $\bar{X}_n$

$$\boxed{\frac{1}{n}\sum_{i=1}^{n} X_i = \bar{X}_n}$$

$$\boxed{\bar{X}_n \xrightarrow{P} \mu_X}$$

$X_1, \ldots, X_n \overset{IID}{\sim}$ some dist, $(\sigma_X^2 < \infty)$

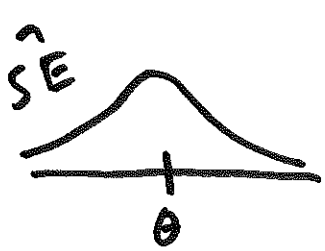
(summary of dist)

unknown parameter θ of interest

suppose you can find an estimator $\hat{\theta}_n$ of

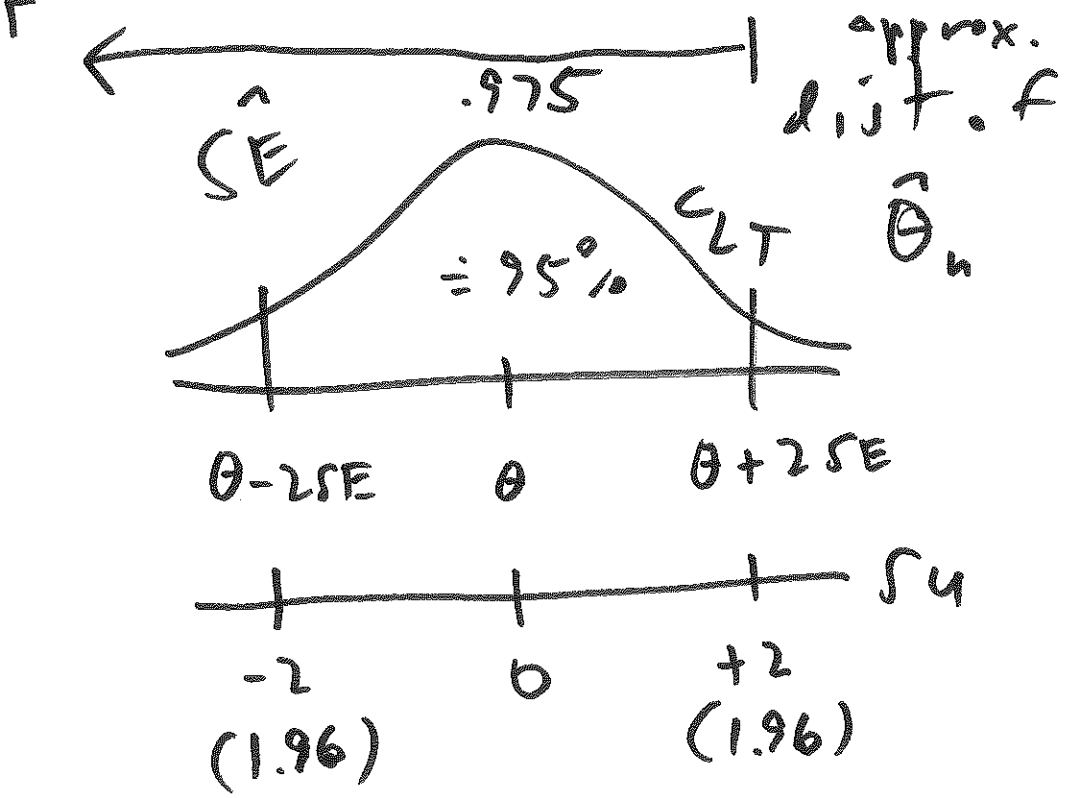
(a function of the X_i)

θ such that



(PDF) dist. of $\hat{\theta}_n$

n large



Neyman was a rabid frequentist: for him θ is a fixed unknown constant and $\hat{\theta}_n$ is a r.v.

fixed r.v. fixed

$$P_F\left(\theta - 2\hat{SE} < \hat{\theta}_n < \theta + 2\hat{SE}\right) \doteq 95\%$$

frequentist

This is not yet what people need; they need something like $P_B\left(A < \theta < B \,/\, \text{data}\right) \doteq .95$

if A, B are fixed

$$P_F(A < \theta < B) = \text{undefined}$$

fixed known, fixed unknown, fixed known

Neyman's confidence trick

$$.95 = P_F\left(\theta - 2\hat{SE} < \hat{\theta}_n < \theta + 2\hat{SE}\right)$$

rewrite $\hat{\theta}_n < \theta + 2\hat{SE} \leftrightarrow$

$\hat{\theta}_n - 2\hat{SE} < \theta$

rewrite $\hat{\theta}_n > \theta - 2\hat{SE} \leftrightarrow$

$\theta < \hat{\theta}_n + 2\hat{SE}$

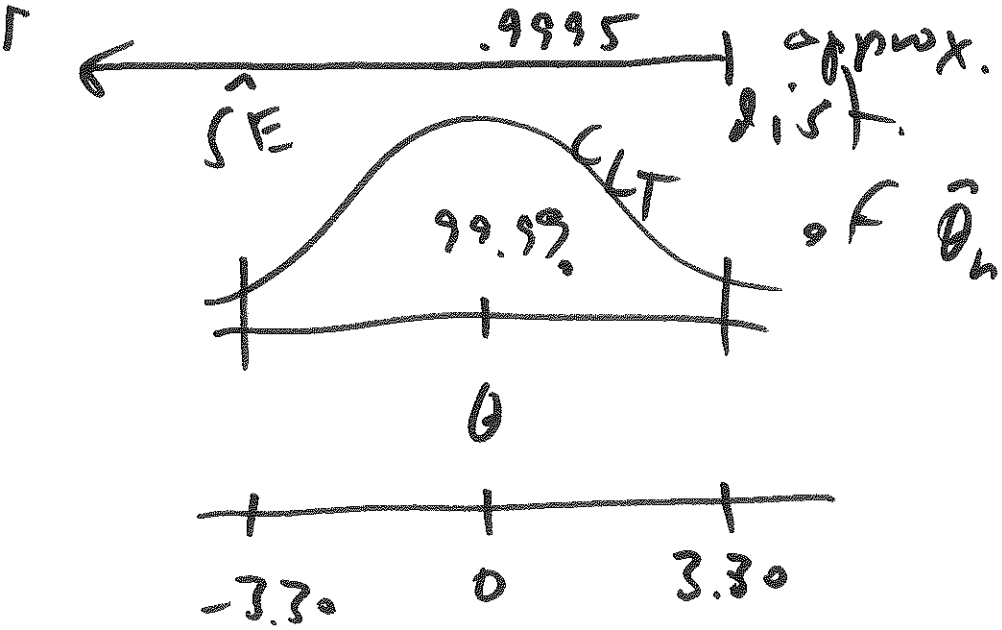
r.v. r.v.

$$.95 = P_F\left(\underbrace{\hat{\theta}_n - 2\hat{SE}}_{A} < \theta < \underbrace{\hat{\theta}_n + 2\hat{SE}}_{B}\right)$$

A, B r.v.s

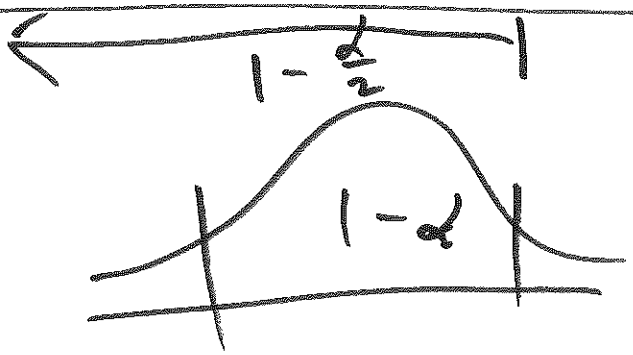
Therefore let's agree to call

$\boxed{\hat{\theta}_n \pm 2\hat{SE}}$ an $\boxed{\text{95\% approximate confidence interval for } \theta \text{ (CI)}}$



An approx. 99.99% CI for θ is

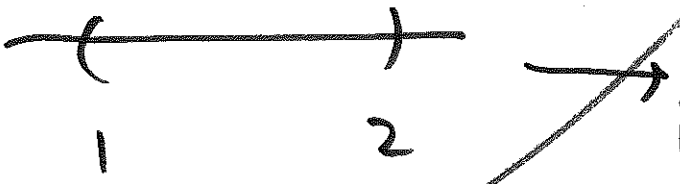
$$\hat{\theta}_n \pm 3.3\,\hat{SE}$$



An approx. $100(1-\alpha)\%$ CI is

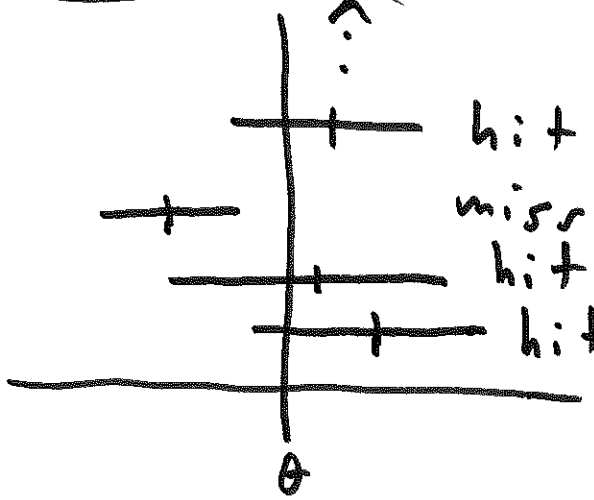
$$\hat{\theta}_n \pm \Phi^{-1}\left(1-\frac{\alpha}{2}\right)\cdot \hat{SE}$$

95% CI for θ



$P_F(1 < \theta < 2) \doteq \cancel{.95}$ is undefined

(95% CIs)



$P_F(\text{hit}) \doteq .95$

= includes θ

Every estimator $\hat{\theta}_n$ is a r.v.

so it has a variance $V(\hat{\theta}_n)$;

$$SE \triangleq \sqrt{V(\hat{\theta}_n)}$$

unfortunately $\sqrt{V(\hat{\theta}_n)}$ often involves θ

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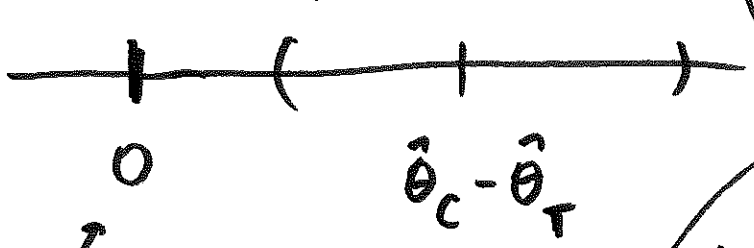
when this occurs, define $\hat{SE}$ to be $\sqrt{\hat{V}(\theta_n)}$

stick in $\hat{\theta}_n$ wherever θ appears

(10.43)



when 0 is not in CI, diff. is (statsig) statistically significant