Definition: An experiment \( E \) is a data-generating process in which all possible outcomes can be listed before \( E \) is performed.

Definition: An event \( E \) is a set of possible outcomes of an experiment \( E \).

Example: Tay-Sachs disease

\( E = \) (the process by which the husband & wife end up with 5 children, each a T-S baby or not) \( \{ \) the \( E \) of interest \( \} = \{ \text{at least 1 T-S baby} \} \)
Definition: The sample space $S$ is the set of all possible outcomes of an experiment $E$. Example: $\{T, S\}$

Let $T$ = "(f)" and $N$ = "(not f)" baby.

Here $S = \{NNNNN, \ldots, TTTTT\}$

Since there are 2 possibilities for each baby $(T, N)$ and 5 babies, the number of elements in $S$ is $2^5 = 32$.

$S$ is an example of a product space:

$\{T, N\} \times \{T, N\} \times \ldots \times \{T, N\} = \{T, N\}^5$. 
Here $E = \{ \text{TNNNN, ..., TTTTT} \}$.

**Notation**

Use $s$ to stand for each individual outcome (elements) of $\mathcal{S}$.

The theory of probability we'll look at in this class was developed by Kolmogorov (1933) in an attempt to rigorize the hypothetical process of throwing a dart at a Venn diagram.

The rules of this dart-throwing were simple: 0) the dart must land somewhere inside (or on the boundary of) the rectangle $\mathcal{S}$, which

\[ A \cup A^c = \mathcal{S} \]
Symbolically stands for the sample space, and all the points where the dart might land in \( S \) are "equally likely" primitive (as yet, an undefined concept).

**Definition**

The complement \( A^c \) of a set \( A \) in \( S \) is the set that contains all elements of \( S \) not in \( A \).

(You can see from the Venn diagram on p. 6 that the dart has to fall either in \( A \) or in \( A^c \), which we could also call \( \text{not} \ A \).) Notation: \( i \) is an element of \( S \) means that \( \{ \text{outcome} \} \) belongs to \( S \).
Definition: A set $A$ is contained in another set $B$, (write $A \subseteq B$) if every element of $A$ is also in $B$; we can also say that $B$ contains $A$ ($B \supseteq A$).

Evidently, if $A$ and $B$ are events,

$A \subseteq B \iff$ (if and only if) if $A$ occurs then so does $B$.

(Theorem) Consequences: If $A$, $B$, $C$ are events then (a) $A \subseteq B$ and $B \subseteq A \iff A = B$

and (b) $A \subseteq B$ and $B \subseteq C \implies A \subseteq C$.

Definition: The cardinality of a set $A$ (written $|A|$) is the number of distinct elements in $A$. 
Example (Tao-Sachs)  \( |S'| = 32 \) (see 10)

**Definition** The set of all subsets of a given set \( S \) is called the **power set** of \( S \), denoted by \( 2^S \); this notation was chosen because, if \( |S| = n \), then \( |2^S| = 2^n \) (in other words, if \( S \) has \( n \) distinct elements then there are \( 2^n \) distinct subsets of \( S \).

**Definition** It's convenient to have a symbol for the set that has no elements in it: \( \emptyset \), the **empty set**.
Example: If $S = \{a, b, c\}$ then $|S| = 3$ and the power set has $2^3 = 8$ sets in it. (Sample space)

Given any set $S$, Kolmogorov (1933) wanted to be able to define probabilities in a logically internally consistent manner (in other words, free from contradictions or paradoxes) to all of the sets in $2^S$.

If $|S|$ is finite, it turns out that nothing nasty can happen.
But if $|\mathbb{I}|$ is infinite, nasty things can unfortunately happen. 

Definition

A set with an infinite number of distinct elements is called an infinite set.

Definition

If the elements of an infinite set $A$ can be placed in 1-to-1 correspondence with the positive integers $\mathbb{N} = \{1, 2, 3, \ldots\}$, $A$ is said to be countably infinite.

Example

The rational numbers are those real numbers that can be expressed as ratios of integers (ex. $\frac{1}{2}, \frac{14}{13}, -\frac{09}{212} \ldots$)
It might seem that there are a lot more rational numbers than integers, but Cantor (1878) showed that the rational numbers are countable. He also showed something even more surprising: the number of distinct values on the real number line is an order of infinity greater than the number of integers or rationals.

Definition: An infinite set that is not countable is called uncountable.

Example: \( \mathbb{N} = \{1, 2, 3, \ldots\} \) is countable, but \( \mathbb{R} = \{\text{all real numbers}\} \) is uncountable.
The mathematical foundation Kolmogorov chose for his development of probability theory is a part of mathematics called measure theory: an attempt to make rigorous the informal concepts of length, area and volume introduced by ancient Greek mathematicians including Euclid (about 2300 years ago) and Pythagoras (about 2500 years ago). However, in the early 1900s people discovered that infinity is a weird thing when you try to make an idea like volume of a sphere in 3-dimensional space rigorous.